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Dipole transitions and Stark effect in the charge-dyon system

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Abstract

We consider the dipole transitions and the linear and quadratic Stark effects in the MICZ-Kepler system interpreted as a charge-dyon system. We show that while the linear Stark effect in the ground state is proportional to the azimuth quantum number (and to the sign of the monopole number), the quadratic Stark effect in the ground state is independent of the signs of the azimuth and monopole numbers.

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1. Introduction

The integrable MICZ-Kepler system suggested independently by Zwanziger [1] and by McIntosh and Cisneros [2] is defined by the following Hamiltonian[5]

$$\mathcal{H}_{\mathrm{MIC}} = \frac{\pi^2}{2} + \frac{s^2}{2r^2} - \frac{1}{r}, \qquad \text{where} \quad [\pi_i, \pi_j] = -s\frac{\varepsilon_{ijk}x_k}{r^3}, \qquad [\pi_i, x_j] = -\imath\delta_{ij}. \tag{1.1}$$

Its distinguished peculiarity is the closed similarity with the Coulomb problem, which insists on the existence of a hidden symmetry given by the angular momentum operator **L** and by the analogue of the Runge–Lenz vector, which are defined by the expressions

$$\mathbf{L} = \mathbf{r} \times \pi + s\frac{\mathbf{r}}{r}, \qquad \mathbf{I} = \frac{1}{2}[\pi \times \mathbf{L} - \mathbf{L} \times \pi] + \frac{\mathbf{r}}{r}. \tag{1.2}$$

This hidden symmetry exists due to the existence of the specific centrifugal term $s^2/2r^2$ in the Hamiltonian. The necessity of incorporation of this term, in the Hamiltonian, describes the

[5] In this paper we use the *Coulomb units*, where the mass, length and time are measured in $\mu, \hbar^2/\mu\gamma, \hbar^3/\mu\gamma^2$ respectively [4]. (Here μ is the mass of the particle, and γ is a Coulomb coupling constant. We restrict ourselves to positive γ corresponding to the attractive MC-Kepler system.) Particularly, the energy unit is $\mu\gamma^2/\hbar^2$.

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motion of the electrically charged non-relativistic particle in the field of Dyrac-dyon (particle that carrieds both electric and magnetic charges). However, it was realized decades ago that for the consistent consideration of the particle–monopole systems, this term should be taken into account. Probably, first time it was pointed out by Zwanziger [1] and Schwinger [3]. Note also that this term appears, when we try to obtain the MICZ-Kepler system, similar to the Coulomb system, from a four-dimensional oscillator [5]. Let us mention, in this respect, that the Schrödinger equation for the MICZ-Kepler system is equivalent to the Schrödinger equation for the system of two well-separated BPS monopoles/dyons (which possess the Coulomb symmetry) [6]. The actual observable difference of the MICZ-Kepler system from the Coulomb problem insists on the change of the range of the total angular momentum from $l = 0, 1, \ldots$ to $l = |s|, |s| + 1, \ldots$ (where the monopole number s takes (half-)integer values), which leads to the $(2|s| + 1)$-fold degeneracy of the ground state with respect to the azimuth quantum number. On the other hand, the lifting of the low bound of the total angular momentum (and degeneracy of the ground state) is the general feature of the quantum-mechanical systems with monopoles. Another general feature of the quantum-mechanical systems with monopoles is the change of the selection rules for the dipole transitions. Namely, while in conventional quantum-mechanical (spherically symmetric) models the selection rules are given by the expressions $m' = m, l' = l - 1; \quad m' = m \pm 1, l' = l \pm 1; m' = m \pm 1, l' = l \mp 1$; in the charge-dyon system other transitions are also possible (see [7] and the references in [8]): $m' = m, l' = l \quad m' = m \pm 1, l' = l$. The specific effect of the choice of the Coulomb potential is hidden symmetry, essentially simplifying the analyses of the system. For example, it makes possible the separation of variables in a few coordinate systems [9].

It seems that the MICZ-Kepler system could be useful for the modification of the existing models of quantum dots related to the Coulomb problem, providing them with a degenerate ground state. Moreover, the search for the MICZ-Kepler system (as well as for the other quantum-mechanical systems with monopoles) in the condensed matter seems to be even more motivated than in high-energy physics and quantum field theory. Indeed, monopoles (and dyons) remain to be hypothetic particles, though their existence is admitted in modern field theoretical models. While in the condensed matter the particle–monopole configuration could be viewed as a short-distance approximation of the behaviour of a charged particle in the vicinity of the pole of magnet. Note that some attempt to incorporate the Dirac monopole into the quantum dot models has been made in [10], giving the satisfactory interpretation of the experimental data; while the MIC-Kepler (charge-dyon) system in the spherical quantum well has been considered in the paper [11]. Naively, one could expect that nth energy level of the MIC-Kepler system should be identical with the $(n + |s|)$th energy level of the Coulomb problem. However, the linear Stark effect in the MICZ-Kepler system (interpreted as a charge-dyon system) completely removes the degeneracy of the energy levels in the charge-dyon system on the azimuth quantum number, in contrast with a hydrogen atom [12]. Thus, one can believe that other observable differences between the MICZ-Kepler and Coulomb systems could also arise due to interaction with external fields.

In this paper, we study other possible differences in the behaviour of the MICZ-Kepler system and Coulomb system, which could affect the condensed matter applications.

At first, we consider, for the completeness, the dipole transitions in the MICZ-Kepler system generated by the planar monochromatic electromagnetic wave. Their difference from the dipole transitions in the so-called dyogen atom (described by the MICZ-Kepler Hamiltonian minus $s^2/2r^2$ term) [7] is in the value of the unessential constant.

Then we consider the Stark effect in the charge-dyon system: in contrast with dipole transitions, the specific choice of potential is important in this consideration. Particularly, since the MICZ-Kepler system admits separation of variables in parabolic coordinates, both linear and quadratic Stark effects could be calculated without any efforts. In addition to the linear Stark effect calculated in [12], we calculate the quadratic one, and specify them for the ground state of the MICZ-Kepler system. We find that the ground state possesses both linear and quadratic Stark effects. The ground energy correction due to linear Stark is proportional to the azimuth quantum number and to the sign of the monopole number, while the quadratic Stark effect is independent of the signs of the monopole and azimuth quantum numbers.

2. Dipole transitions

Let us consider the dipole transitions in the MICZ-Kepler system interacting with a planar monochromatic electromagnetic wave, which are completely similar to those in the 'dyogen atom' [7].

The wave is defined by the vector potential:

$$\mathbf{A} = A_0 \mathbf{u} \cos(\omega t - \mathbf{kr}), \qquad \nabla \cdot \mathbf{A} = 0 \tag{2.1}$$

where $\mathbf{u}$ is the polarization vector, $\mathbf{uk} = 0$. Assuming that the magnitude of this field is small enough, we could represent the interaction energy as follows:

$$\mathcal{H} = \frac{(\boldsymbol{\pi} - \mathbf{A})^2}{2} + \frac{s^2}{2r^2} - \frac{1}{r} \approx \mathcal{H}_{\mathrm{MIC}} - \mathbf{A}\boldsymbol{\pi}, \tag{2.2}$$

where $\mathcal{H}_{\mathrm{MIC}}$ is defined by (1.1).

For the calculation of the matrix element of dipole transitions we shall use the wavefunctions of the non-perturbed MICZ-Kepler system in the spherical coordinates, which are the solutions of the following spectral problem:

$$\mathcal{H}_{\mathrm{MIC}}\psi = \mathcal{E}_{(0)}\psi, \qquad \mathbf{L}^2\psi = l(l+1)\psi, \qquad L_3\psi = m\psi. \tag{2.3}$$

These wavefunctions are given by the expressions:

$$\psi_{nlm}(\mathbf{r}; s) = c_{nl} r^l \,\mathrm{e}^{-r/n} F\left(l - n + 1, 2l + 2, \frac{2r}{n}\right) d^l_{ms}(\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}. \tag{2.4}$$

Here d^l_{ms} is the Wigner d-function, the energy spectrum is defined by the expression $\mathcal{E}_0 = -1/2n^2$, and the quantum numbers n, l, m have the following ranges of definition:

$$n = l+1, l+2, \ldots, \qquad l = |s|, |s+1|, \ldots, \qquad m = -l, -l+1, \ldots, l-1, l. \tag{2.5}$$

The normalization constant c_{nl} is given by the expression (see, e.g., [9])

$$c_{nl} = \frac{2^l}{n^{l+2}(2l+1)!}\sqrt{\frac{(2l+1)(n+l)!}{\pi(n-l-1)!}}. \tag{2.6}$$

The matrix element for dipole transitions in the long wave approximation looks as follows [13]

$$M_{n,l,m|n',l',m'} = -\left[\frac{2\pi N}{V\omega}\right]^{1/2} \mathbf{u}\langle n, l, m|\boldsymbol{\pi}|n', l', m'\rangle, \tag{2.7}$$

where N is the density of photons in the volume V.

Taking into account the commutation relation $[\mathbf{r}, \mathcal{H}_{\mathrm{MIC}}] = \imath\boldsymbol{\pi}$, one can represent the probability of transition from the state (n, l, m) to the state (n', l', m') in the unit time in the following form:

$$\mathrm{d}w_{n,l,m|n',l',m'} = \frac{N\omega^3}{2\pi}|\mathbf{ud}_{n,l,m|n',l',m'}|^2\,\mathrm{d}\Omega,$$
$$\text{where}\quad \mathbf{d}_{n,l,m|n',l',m'} = (\mathcal{E}_{n',l',m'} - \mathcal{E}_{n,l,m})\langle n, l, m|\mathbf{r}|n', l', m'\rangle. \tag{2.8}$$

A straightforward calculation yields the result

$$
\begin{aligned}
\mathbf{ud}_{n,l,m|n',l',m'} = I(n,l|n',l')\Bigg[\frac{u_x+\imath u_y}{2}\Bigg(&\frac{l+1}{2(2l+1)}\sqrt{(l+m)(l^2-s^2)}\delta_{m-1|m'}\delta_{l-1|l'}\\
&+\frac{\sqrt{(l+1)(l-m+1)(l-m+2)((l+1)^2-s^2)}}{2\sqrt{l+2}(l+1)(2l+2)}\delta_{m-1|m'}\delta_{l+1|l'}\\
&+s\frac{\sqrt{(l-m+1)(l+m)}}{l(l+1)}\delta_{m-1|m'}\delta_{l|l'}\Bigg)\\
&-\frac{u_x-\imath u_y}{2}\Bigg(\frac{l+2}{2(2l+3)}\sqrt{(l+m+2)((l+1)^2-s^2)}\delta_{m+1|m'}\delta_{l+1|l'}\\
&-\sqrt{\frac{l}{l+1}}\frac{\sqrt{(l-m-1)(l-m)(l^2-s^2)}}{l(2l-1)}\delta_{m+1|m'}\delta_{l-1|l'}\\
&+s\frac{\sqrt{(l+m+1)(l-m)}}{l(l+1)}\delta_{m+1|m'}\delta_{l|l'}\Bigg)\\
&+u_z\Bigg(\frac{\sqrt{(l+1)(l^2-m^2)(l^2-s^2)}}{\sqrt{l}l(2l+1)}\delta_{l-1|l'}\\
&+\frac{\sqrt{(l+1)((l+1)^2-m^2)((l+1)^2-s^2)}}{\sqrt{l+2}(l+1)(2l+1)}\delta_{l+1|l'}+s\frac{m}{l(l+1)}\delta_{l|l'}\Bigg)\delta_{m|m'}\Bigg]
\end{aligned}
\tag{2.9}
$$

where

$$
\begin{aligned}
I(n,l|n',l') = \int_0^\infty c_{nl}c^*_{n'l'}r^{l+l'}\,\mathrm{e}^{-(\frac{r}{n'}+\frac{r}{n})}F\left(l-n+1,2l+2,\frac{2r}{n}\right)\\
\times F\left(l'-n'+1,2l'+2,\frac{2r}{n}\right)r^3\,\mathrm{d}r,
\end{aligned}
\tag{2.10}
$$

and c_{nm} is defined by (2.6).

It is seen that the presence of Dirac monopole changes the selection rules of the system. Namely, in the absence of monopole one has

$$u_z \neq 0 : m = m', l' = l - 1 \tag{2.11}$$

$$|u_x + \imath u_y| \neq 0 : m' = m \pm 1, l' = l \pm 1; \quad m' = m \pm 1, l' = l \mp 1. \tag{2.12}$$

In the presence of Dirac monopole, when $s \neq 0$ other transitions are also possible [7, 14]:

$$u_z \neq 0 : m = m', l' = l \tag{2.13}$$

$$|u_x + \imath u_y| \neq 0 : m' = m \pm 1, l' = l. \tag{2.14}$$

So, the presence of monopole makes the selection rules less rigorous. Namely, besides (2.12), the transitions preserving the orbital quantum number l are also allowed, (2.14). When the electromagnetic wave has transversal polarization ($u_z = 0$), the transitions preserving the orbital quantum number, and changing the azimuth quantum number, become possible. When longitudal mode in the electromagnetic wave appears ($u_z \neq 0$), the transitions, preserving both orbital and azimuth quantum numbers, are also admissible.

3. The Stark effect

The Hamiltonian of the MICZ-Kepler system (interpreted as a charge-dyon system) in the external constant uniform electric field is of the form

$$\mathcal{H}_{\text{Stark}} = \mathcal{H}_{\text{MIC}} + \mathbf{Er}. \tag{3.1}$$

Similar to the Coulomb system in the constant uniform magnetic field, this system possesses two constants of motion:

$$J \equiv \mathbf{n}_E\mathbf{L}, \qquad I = \mathbf{n}_E\mathbf{I} + \frac{|\mathbf{E}|}{2}(\mathbf{n}_E \times \mathbf{r})^2, \tag{3.2}$$

where $\mathbf{n}_E = \mathbf{E}/|\mathbf{E}|$ is the unit vector directed along the external electric field, and $\mathbf{L}$ and $\mathbf{I}$ are given by expressions (1.2). While the origin of the first constant of motion is obvious, the validity of the second expression can be checked by the straightforward calculation. Due to the existence of the second constant of motion, the charge-dyon system interacting with the external electric field admits the separation of variables in parabolic coordinates. As a consequence, similar to the hydrogen atom, one can calculate the quadratic Stark effect in the charge-dyon system [4]. We assume that the electric field $\mathbf{E}$ is directed along the positive x_3-semiaxes, and the force acting on the electron is directed along the negative x_3-semiaxes. We represent the momentum operator $\boldsymbol{\pi}$ as follows:

$$\boldsymbol{\pi} = -\imath\nabla - s\mathbf{A}_D, \qquad \mathbf{A}_D = \frac{1}{r(r - x_3)}(x_2, -x_1, 0) \tag{3.3}$$

where $\mathbf{A}_D$ is the potential of the Dirac monopole with the singularity line directed along the positive semiaxis x_3. Choosing the parabolic coordinates $\xi, \eta \in [0, \infty)$, $\varphi \in [0, 2\pi)$ defined by the formulae

$$x_1 + \mathrm{i}x_2 = \sqrt{\xi\eta}\,\mathrm{e}^{\mathrm{i}\varphi}, \qquad x_3 = \tfrac{1}{2}(\xi - \eta). \tag{3.4}$$

and making the substitution

$$\psi(\xi, \eta, \varphi) = \Phi_1(\xi)\Phi_2(\eta)\frac{\mathrm{e}^{\mathrm{i}m\varphi}}{\sqrt{2\pi}}. \tag{3.5}$$

we separate the variables in the Schrödinger equation for the Hamiltonian (3.1), and arrive at the system [9]

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\frac{\mathrm{d}\Phi_1}{\mathrm{d}\xi}\right) + \left[\frac{\mathcal{E}}{2}\xi - \frac{|\mathbf{E}|}{4}\xi^2 - \frac{(m+s)^2}{4\xi}\right]\Phi_1 = -\beta_1\Phi_1, \\ &\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta\frac{\mathrm{d}\Phi_2}{\mathrm{d}\eta}\right) + \left[\frac{\mathcal{E}}{2}\eta + \frac{|\mathbf{E}|}{4}\eta^2 - \frac{(m-s)^2}{4\eta}\right]\Phi_2 = -\beta_2\Phi_2, \qquad \beta_1 + \beta_2 = 1. \end{aligned} \tag{3.6}$$

It is seen that $\beta_1 - \beta_2$ is the eigenvalue of the operator I in (3.2).

For $s = 0$ these equations coincide with the similar equations for the hydrogen atom in the parabolic coordinates [4]. Hence, similar to that, we can consider the energy $\mathcal{E}$ as a fixed parameter, and $\beta_{1,2}$ as the eigenvalues of the corresponding operators. These quantities are defined after solving the above equations, as the functions of $\mathcal{E}$ and $\mathbf{E}$. Then, due to the relation $\beta_1 + \beta_2 = 1$, the energy $\mathcal{E}$ becomes a function of the external field $\mathbf{E}$. Let us consider the terms containing the electric field $|\mathbf{E}|$ as a perturbation. Thus, in zero approximation ($\mathbf{E} = 0$) we get

$$\Phi_1 = \sqrt{\kappa}\,\Phi_{n_1,m+s}(\sqrt{\kappa}\xi), \qquad \Phi_1 = \sqrt{\kappa}\,\Phi_{n_2,m-s}(\sqrt{\kappa}\eta). \tag{3.7}$$

Here

$$\Phi_{pq}(x) = \frac{1}{|q|!}\sqrt{\frac{(p+|q|)!}{p!}}\,\mathrm{e}^{-x/2}(x)^{|q|/2}{}_1F_1(-p; |q|+1; x). \tag{3.8}$$

and n_1, n_2 are non-negative integers

$$\beta_1^{(0)} = \left(n_1 + \frac{|m_1|+1}{2}\right)\kappa, \qquad \beta_2^0 = \left(n_2 + \frac{|m_2|+1}{2}\right)\kappa, \tag{3.9}$$

and

$$\kappa = \sqrt{-2\mathcal{E}}, \qquad m_a = m - (-1)^a s \qquad a = 1, 2. \tag{3.10}$$

It is seen from the above expressions that the calculation of the first- and second-order corrections to $\beta_{1,2}^{(0)}$ will be completely similar to those in the Coulomb problem [4], if one replaces $|m| \to |m+s|$ in β_1, Φ_1, and $|m| \to |m-s|$ in β_2, Φ_2. These substitutions yield the following expressions:

$$\beta_a^{(1)} = -\frac{(-1)^a|\mathbf{E}|}{4\kappa^2}\left(6n_a^2 + 6n_a|m_a| + m_a^2 + 6n_a + 3|m_a| + 2\right) \tag{3.11}$$

$$\beta_a^{(2)} = -\frac{|\mathbf{E}|^2}{16\kappa^5}(|m_a| + 2n_a + 1)\left(4m_a^2 + 17\left(2|m_a|n_a + 2n_a^2 + |m_a| + 2n_a\right) + 18\right). \tag{3.12}$$

Then we get

$$\beta_1^0 + \beta_2^{(0)} = \kappa n, \qquad \beta_1^{(1)} + \beta_2^{(1)} = \frac{3|\mathbf{E}|}{2\kappa^2}A, \qquad \beta_1^{(2)} + \beta_2^{(2)} = -\frac{|\mathbf{E}|^2}{16\kappa^5}B, \tag{3.13}$$

where we introduce the notations

$$A \equiv nn_- - \frac{ms}{3}, \qquad B \equiv 17n^3 - 3nn_-^2 + 54An_- + 19n - 9n(m^2 + s^2), \tag{3.14}$$

and the quantum numbers

$$n = n_1 + n_2 + \frac{|m+s| + |m-s|}{2} + 1, \qquad n_- \equiv n_1 - n_2 + \frac{|m+s| - |m-s|}{2}. \tag{3.15}$$

Taking into account that $\beta_1 + \beta_2 = 1$, we get

$$\kappa n + \frac{3|\mathbf{E}|A}{2\kappa^2} - \frac{|\mathbf{E}|^2 B}{16\kappa^5} = 1. \tag{3.16}$$

Iteratively solving this equation, we get

$$\kappa = \kappa_0 + |\mathbf{E}|\kappa_1 + |\mathbf{E}|^2\kappa_2, \qquad \kappa_0 = \frac{1}{n}, \qquad \kappa_1 = -\frac{3An}{2}, \qquad \kappa_2 = n^3\left(\frac{Bn}{16} - \frac{9A^2}{2}\right). \tag{3.17}$$

Then, from $E = -\kappa^2/2$ we find the energy of the system

$$\mathcal{E} = -\frac{1}{2n^2} + \frac{3|\mathbf{E}|}{2}\left(nn_- - \frac{ms}{3}\right) - \frac{|\mathbf{E}|^2 n^2}{16}(17n^4 - 3(nn_- - 3ms)^2 - 9n^2m^2 + 19n^2 - 9n^2s^2 + 21(ms)^2). \tag{3.18}$$

One can represent the quantum numbers (3.15) as follows:

$$n = \begin{cases} n_1 + n_2 + |s| + 1 & \text{for} \quad |m| \leqslant |s| \\ n_1 + n_2 + |m| + 1 & \text{for} \quad |m| > |s|, \end{cases} \qquad n_- = \begin{cases} n_1 - n_2 + m\,\mathrm{sgn}\,s & \text{for} \quad |m| \leqslant |s| \\ n_1 - n_2 + s\,\mathrm{sgn}\,m & \text{for} \quad |m| > |s|. \end{cases} \tag{3.19}$$

The ground state of the non-perturbed charge-dyon system corresponds to the following values of quantum numbers: $n_1 = n_2 = 0$, $|m| \leqslant |s|$. Hence,

$$n = |s| + 1, \qquad n_- = m\,\mathrm{sgn}\,s, \qquad m = -|s|, -|s| + 1, \ldots, |s| - 1, |s|. \tag{3.20}$$

Substituting these expressions in (3.18), we get

$$\mathcal{E}_0 = -\frac{1}{2(|s|+1)^2} + m\,\mathrm{sgn}\,s|\mathbf{E}|\left(|s|+\frac{3}{2}\right) - \frac{|\mathbf{E}|^2(|s|+1)^2}{16}[17(|s|+1)^4 + (|s|+1)^2(19-9s^2) - 6m^2(|s|+2)]. \quad (3.21)$$

It is seen that the ground state of the non-perturbed charge-dyon system has $(2|s|+1)$-fold degeneracy (by the azimuth quantum number m), while the linear Stark effect completely removes the degeneracy on m. It is proportional to the azimuth quantum number m, while its sign depends on the relative sign of the monopole numbers s and m (the linear Stark effect in the 'dyogen atom' possesses similar properties [14]). In contrast to the linear Stark effect, the quadratic Stark effect of the ground state is independent neither of sign s, nor of sign m.

4. Conclusion

We have studied the behaviour of the MICZ-Kepler system, interpreted as a charge-dyon system, in the external fields, and for testing its differences from the hydrogen atom.

We considered the dipole transitions in the MICZ-Kepler system interacting with a planar monochromatic electromagnetic wave. Similar to the other spherically-symmetric systems with monopole, the MICZ-Kepler system admits dipole transitions which are forbidden in conventional quantum mechanics. Namely, in conventional quantum mechanics the dipole transitions should satisfy the selection rules (2.12); while in the systems with monopole, the transitions satisfying the selection rules (2.14) are also possible. The behaviour of the MICZ-Kepler system in the constant electric field is also different from that in the hydrogen atom. In the earlier work, it was observed that the linear Stark effect completely removes the degeneracy of the spectrum in the MICZ-Kepler system [12]. In the present paper, we calculated the quadratic Stark effect too. Considering the ground state of the MICZ-Kepler charge-dyon system (which has $(2|s|+1)$-fold degeneracy), we found that the linear Stark effect is proportional to the azimuth quantum number, and to the sign of the monopole number as well; while the quadratic Stark effect depends on the absolute values of the azimuth and monopole numbers.

Let us note that besides the system of two well-separated BPS dyons [6], other integrable generalizations of the MICZ-Kepler system on the curved spaces also exist [15]. In the context of quantum dots application, these systems could be viewed as a model with position-dependent effective mass. Clearly, the dipole transitions in these systems would be similar to those in conventional charge-dyon systems, but the Stark effect could be essentially different. We will consider it elsewhere. Also, we expect, from some preliminary analyses, that similar consideration could be performed after incorporating the monopole into the more complicated integrable system considered in [16]. This system also includes, besides the Coulomb potential, the oscillator one and the constant magnetic field. The present study has been performed for testing the possible consequences of incorporation of the MICZ-Kepler system in the models of quantum dot. From this viewpoint, MICZ-Kepler-like generalization of the system [16] seems to be especially interesting.

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